frequency identification, software, control theory

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FINITE-FREQUENCY IDENTIFICATION: THE SOFTWARE IMPLEMENTATION IN THE GAMMA SYSTEM

The finite-frequency identification method was designed for the needs of active identification. The test signal is represented by the sum of harmonics with automatically tuned (self-tuned) amplitudes and frequencies where the number of harmonics does not exceed the state space dimension of the plant. The self-tuning of amplitudes is carried out to satisfy those requirements on the bounds of the input and output which hold true in the absence of a test signal. In this paper the software implementation of finite-frequency identification method in the system GAMMA is considered. GAMMA is a two-level CACSD tool for identification and controllers algorithms synthesis for linear plants.

1. INTRODUCTION

At present, the control theory has at its disposal a number of identification methods for plants specified by linear differential equations. Conventionally, these methods fall into two categories depending on the assumptions on the measurement errors and exogenous disturbances affecting the plant.

The methods of the first class deal with the plants subjected to disturbances of the stochastic nature; i.e., random processes having known statistical characteristics. These are various versions of the method of least squares and the stochastic approximation method; e.g., see well-known monographs [1].

The second class comprises the identification methods under unknown-but-bounded disturbances (whose statistical properties are not known) such as randomized algorithms [2] and finite-frequency identification, see [3].

A somewhat specific position is occupied by the method of instrumental variables [4]. It is developed in the framework of the first class; however, in contrast to the other methods in the class, it is applicable to the problems of the second class so that it is reasonable to consider it as a method of the second category.

The identification process can have passive or active forms. In the *passive* identification, the measured input to the plant has the meaning of a control action which depends on the control

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objectives and is not related to identification of the plant. With such an input, identification might not be possible; hence, *active* identification is often practiced where, in addition to control, the measured input contains an extra component, a so-called test signal aimed at identifying the plant. The finite-frequency identification method was designed for the needs of active identification. The test signal is represented by the sum of harmonics with automatically tuned (self-tuned) amplitudes and frequencies where the number of harmonics does not exceed the state space dimension of the plant. The self-tuning of amplitudes is carried out to satisfy those requirements on the bounds on the input and output which hold true in the absence of a test signal.

The MATLAB-toolbox for finite-frequency identification is described in [5]. In this paper the software implementation of this method in the system GAMMA [6,7] is considered.

MATLAB [8] and GAMMA are oriented on different groups of users. MATALB is intended for the researchers who well know the control theory. The researcher easily creates the program for the solution of real problems of its data domain using a rich spectrum of m-functions. GAMMA is intended for the engineers-developers of a control system. The purposes of this group and a small time for Control System development eliminate a capability of their participation in creation of the software for the solution of their problem. Besides the necessity of a profound knowledge of the theory of control also handicaps their work with the MATLAB.

2. THE IDENTIFICATION PROBLEM

A completely controllable, asymptotically stable plant is described by the following equation:

$$d_n y^{(n)} + \ldots + d_1 \dot{y} + y = k_\gamma u^{(\gamma)} + \ldots + k_1 \dot{u} + k_0 u + f \, , t \geq t_0 \, , \tag{1}$$

where $y(t)$ is the measured output, $u(t)$ is the input to be controlled, $y^{(i)}$, $u^{(j)}$ $(i = \overline{1,n},\, j = \overline{1,\gamma})$ are the derivatives of these functions, $f(t)$ is unknown-but-bounded disturbance. The coefficients d_i and k_j $(i = \overline{1,n},\, j = \overline{0,\gamma})$ are some unknown numbers; n and γ are known, and $\gamma < n.$.

The identification problem is to find estimates $\hat{d}_i$ and $\hat{k}_j$ $(i = \overline{1,n},\, j = \overline{0,\gamma})$ of the plant coefficients such that the identification errors satisfy the following relations:

$$\hat{d}_i \div d_i \leq \varepsilon_i^d \, , \hat{k}_j \div k_j \leq \varepsilon_j^k \quad i = \overline{1,n} \quad j = \overline{0,\gamma} \tag{2}$$

where ε_i^d and ε_j^k $(i = \overline{1,n},\, j = \overline{0,\gamma})$ are given numbers, and the symbol $\div$ means: $a \div b = |a - b| / |b|$ if $b \neq 0$ and $a \div b = |a|$ otherwise.

Let us consider the finite-frequency identification technique, which gives a solution to this problem.

A set of 2*n* numbers

$$\alpha_k = \operatorname{Re} w(j\omega_k), \beta_k = \operatorname{Im} w(j\omega_k) \quad k = \overline{1,n} \tag{3}$$

where

$$w(s) = \frac{k_\gamma s^\gamma + \ldots + k_0}{d_n s^n + d_{n-1} s^{n-1} + \ldots + 1} \tag{4}$$

is called the *frequency domain parameters* (FDP).

The FDP estimates are determined experimentally as follows: after the plant (1) is excited by the test signal

$$u = \sum_{k=1}^{n} \rho_k \sin \omega_k (t - t_0), t \geq t_0 \tag{5}$$

where the amplitudes $\rho_k (k = \overline{1,n})$ and test frequencies $\omega_k (k = \overline{1,n})$ are specified positive numbers, its output is fed to the Fourier filters, whose outputs give the following FDP estimates:

$$\hat{\alpha}_k = \alpha_k(\tau) = \frac{2}{\rho_k \tau} \int_{t_F}^{t_F+\tau} y(t) \sin \omega_k (t - t_0) dt,$$
$$\hat{\beta}_k = \beta_k(\tau) = \frac{2}{\rho_k \tau} \int_{t_F}^{t_F+\tau} y(t) \cos \omega_k (t - t_0) dt, \quad (k = \overline{1,n}) \tag{6}$$

where τ is a filtering time and $t_F \geq t_0$ is the initial instant for filtering.

In order to formulate conditions on the convergence of the FDP estimates (6) to the true FDP, the following functions are introduced:

$$l_k^{\alpha}(\tau) = \frac{2}{\rho_k \tau} \int_{t_F}^{t_F+\tau} \bar{y}(t) \sin \omega_k (t - t_0) dt,$$
$$l_k^{\beta}(\tau) = \frac{2}{\rho_k \tau} \int_{t_F}^{t_F+\tau} \bar{y}(t) \cos \omega_k (t - t_0) dt, \quad (k = \overline{1,n}) \tag{7}$$

where $\bar{y}(t)$ is the "natural" output of the plant when the test signal (5) is absent ($u(t) = 0$).

A disturbance $f(t)$ is called strongly FF-filterability if, for the given numbers δ^{α} and δ^{β}, there exists filtering time τ^* such that

$$\frac{|\, l_k^{\alpha}(\tau^*) \,|}{|\, \alpha_k(\tau^*) \,|} \leq \delta^{\alpha},$$
$$\frac{|\, l_k^{\beta}(\tau^*) \,|}{|\, \beta_k(\tau^*) \,|} \leq \delta^{\beta}, \ (k = \overline{1,n}), \ \tau \geq \tau^*. \tag{8}$$

Conditions (8) can be examined by experiment.

If the disturbance $f(t)$ is strongly FF-filterability, then the filtering errors $\Delta\alpha_k(\tau) = \alpha_k - \alpha_k(\tau)$, $\Delta\beta_k(\tau) = \beta_k - \beta_k(\tau)$ $(k = \overline{1,n})$ have the following properties: $\lim_{\tau\to\infty} \Delta\alpha_k(\tau) = \lim_{\tau\to\infty} \Delta\beta_k(\tau) = 0 \ (k = \overline{1,n})$.

The estimates of the plant coefficients are found on the basis of the FDP estimates. In fact, the identity $w(s)=\dfrac{k(s)}{d(s)}$ and expressions (3) give the following system of the linear algebraic equations:

$$\hat{k}(s_k)-(\alpha_k+j\beta_k)\hat{\hat{d}}(s_k)=\alpha_k+j\beta_k\,,\ (k=\overline{1,n})\ , \tag{9}$$

where $\hat{\hat{d}}(s)=\hat{d}\ (s)-1=\hat{d}_n s^n+...+\hat{d}_1 s\,,\ \hat{k}(s)=\hat{k}_\gamma s^\gamma+...+\hat{k}_0\,,\ s_k\ =\ j\omega_k\ \ (k=\overline{1,n})$.

If the plant (1) is completely controllable then system (9) has a unique solution d_i, k_j $(k=\overline{1,n},\ j=\overline{1,\gamma})$ which does not depend on the choice of the frequencies $\omega_i(\omega_i\neq\omega_j(i\neq j),\omega_i\neq 0(i=\overline{1,n}))$.

Substituting the FDP by their estimates, the following *frequency equations* of identification

$$\hat{k}(s_k)-(\hat{\alpha}_k+j\hat{\beta}_k)\hat{\hat{d}}(s_k)=\hat{\alpha}_k+j\hat{\beta}_k\,,\ (k=\overline{1,n})\ , \tag{10}$$

are obtained.

In order to examine requirements (2), the frequency techniques of model validation [9] may be used.

3. GAMMA SYSTEM FEATURES

GAMMA is a two-level CACSD tool for identification and controllers algorithms synthesis of linear plants by engineers-developers of control system.

The first level (the engineers environment) represents set of design procedures (the directives) that provide the automatic solution of design problems by an engineer-developers of control system. The second level (the researchers environment) intends for development and modernization the software of first level by a researcher.

3.1 THE ENGINEERS ENVIRONMENT

The Engineers environment's basis is a directive. The directive is a program consisting of three parts: a software for user interface, computational part and means for output of intermediate and final results. Each directive decides the definite class of problems of a designing of control algorithm. All directives are subdivided into three classes:

- Synthesis (LQ-optimization, H_∞ - suboptimal control, controller synthesis under the specified tolerance on steady-state errors and etc);
- Identification (finite-frequency identification, identification with selftuning of test signal, identification with selftuning of identification time and etc);
- Adaptive control.

In Engineers environment a user chooses the directive from the directives list; after a request of computer, he enters in natural form the initial data: the differential equations of plant, disturbance boundaries, technical indices tolerances, etc in dependence on class of problem. The dialog form for initial data input is shown in Fig. 1. Then a directive is performed automatically (without user participation). Analyzing the results he makes a decision on an acceptability of outcome (Fig.2).

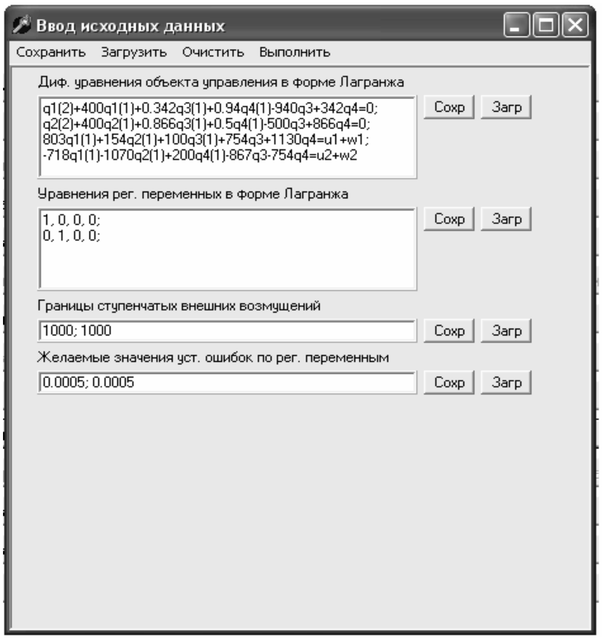

Fig. 1. The dialog form with initial data of directive.

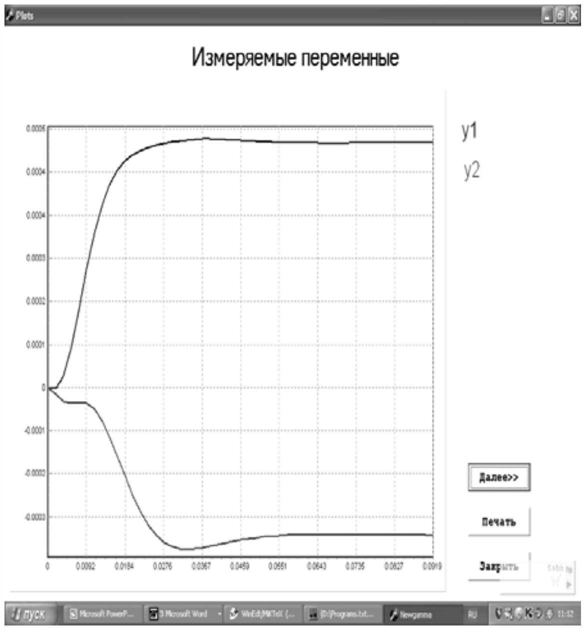

Fig. 2. The plot of control process.

3.2 THE RESEARCHERS ENVIRONMENT

The researchers environment intends for directives development. The directives of GAMMA is developed with use of program modules. Each module solves the elementary problem of control theory (for example, controlability analysis, solving of Riccati equation, etc). The researcher may use any programming language for modules writing, but there are the some rules that he has to observe.

The language INSTRUMENT is used for directives implementation in the GAMMA system. The interpreter of this languages is the part of GAMMA. Before INSTUMENT-1 had the limited set of operators: operators for input and output of data, operator for running of calculation modules, conditional and unconditionally control transfer. So the calculation modules were writing in C and PASCAL.

Now the possibilities of INSTUMENT are substantially increased. All standard operation were added into the INSTRUMENT: loops, conditional, subroutines, operations with arrays and etc. As the result the most of calculation modules are wrote in the INSTRUMENT now. Besides the possibility for using modules written in the another languages (C or PASCAL or the other) is maintained as in the previous version of INSTRUMENT.

4. GAMMA SOFTWARE FOR FINITE-FREQUENCY IDENTIFICATION

The software for finite-frequency identification is implemented as the number of directives. Let’s describe one of them: D123su – the directive of finite-frequency identification where the amplitudes and the frequencies of the test signal and the filtering time are specified numbers.

4.1 THE STRUCTURE OF THE DIRECTIVE D123su

The structure of the directive is the following:

<D123su>=<interface><calculation part><results>

<calc. part>=<Cauchy><PMV1><ABtoFR><Analysis><FourSu> <Freqd>.

The initial data of the directive:

- `d,k,m` – polynomial of the plant;
- `np` – plant order;
- `par` – external disturbance parameters;
- `h` – sampling time,
- `om` – test frequencies;
- `rho` – amplitudes of the test frequencies;
- `Ptau` – identification time (number of periods of the minimal test frequency);
- `Tfi` – start time of filtration (number of periods of the minimal test frequency);
- `x` - initial condition;
- `Tbegin` – start time of simulation.

The results:

- `vdd` – estimates of the denominator coefficients of the plant transfer function;
- `vkd` - estimates of the numerator coefficients of the plant transfer function;

4.2 THE MODULES OF THE DIRECTIVE D123su

The following modules were used in the directive:

Cauchy – conversion of the plant model to the state space form.

Syntax: Cauchy`[d,k,m][A,B,C,D]`,

where `A,B,C,D` – the matrix of state space form;

PMV1 – computation of plant inputs and recalculation of test frequencies

Syntax: `PMV1[om,rho,par,h,Ptau,np,TBegin][t,u1,u,om1]`,

where `t` – time vector, `u,u1` – plant inputs, `om` – new test frequencies.

ABtoFR – computation of matrix of discrete-time model.

Syntax: `ABtoFR[A, B, h][F, R]`,

where `F,R` are the matrix of discrete-time model.

Analysis – plant simulation.

Syntax: `Analysis[F,R,C,D,u1,t,x0][x,y]`,

where `y` is the plant output.

FourSu – computation of the FDP estimates.

Syntax:

`FourSu[y,u,om1,rho,np,h,Ptau,Tfi][valf,vbet]`,

where `valf,vbet` – the estimates of the plant FDP.

Freqd – solution of the frequency equations of identification

Syntax:

`Freqd[om,h,valf,vbet][vkd,vdd]`,

where `vdd` and `vkd` are the estimates of coefficients of discrete-time model of the plant.

5. EXAMPLE

The following plant has been used to test the directive D123su:

$$0.2\dddot{y} + 1.24\ddot{y} + 5.24\dot{y} + y = -0.4\dot{u} + u + f$$

Discrete-time transfer function of this plant under sampling time *h=0.2* is

$$W(z)=\frac{-0.0206z^2+0.0224z+0.0192}{z^3-1.7260z^2+1.0360z-0.2894} \qquad (11)$$

The initial data of directive:

- Polynomial of the plant $\mathrm{d(s)}=0.2\mathrm{s}^3+1.24\mathrm{s}^2+5.24\mathrm{s}+1,\ \mathrm{k(s)}=-0.4\mathrm{s}+1,\ \mathrm{m(s)}=1$.
- Plant order *np=3;*
- External disturbance $f(t)=sign(\sin(2.1t))$;
- Sampling time *h=0.2;*
- Test signal $u(t)=0.3\sin 0.2t+3\sin 4t+3\sin 6t$;
- Identification time *Ptau=3;*
- Start time of filtration *Tfi=1*.

The following results were obtained:

- FDP estimates:
  $\hat{\alpha}=[0.460051\ \ -0.0591760\ \ \ 0.022480]$; $\hat{\beta}=[-0.63288\ \ \ \ 0.0723600\ \ \ \ 0.0533290]$.

- Identified discrete-time model of the plant:

$$W(z)=\frac{-0.0192z^2+0.0203z+0.0218}{z^3-1.7092z^2+1.0009z-0.2713} \qquad (12)$$

The coefficients of model (11) and (12) are sufficiently closed.

REFERENCES

[1] LJUNG L., *System Identification. Theory for the User*, Englewood Cliffs: Prentice-Hall, 1987. Translated under the title Identifikatsiya sistem. Teoriya dlya pol'zovatelya, Moscow: Nauka, 1991.
[2] GRANICHIN O.N. AND POLYAK B.T., *Randomized Algorithms of Estimation and Optimization under Almost Arbitrary Noise*, Moscow: Nauka, 2003.
[3] ALEXANDROV A.G., *Finite-frequency identification: selftuning of test signa*l. Preprints of the 16th IFAC World Congress ,CD-ROM, 2005.
[4] WONG, K.Y. AND POLAK E., *Identification of linear discrete time systems using instrumental variable method.* IEEE Trans. Automat. Control. Vol. AC-12., 1967, pp. 707-718.
[5] ALEXANDROV A.G., ORLOV U.F., MIKHAYLOVA L.S., *ADAPLAB-M: identification and adaptation toolbox for MATLAB.* 13th Symposium on System Identification. IFAC SYSID'03, 2003, pp. 995-1000.
[6] ALEXANDROV A.G., ISAKOV R.V., MIKHAYLOVA L.S., *The structure of software for automatic control algorithm development.* The Automation and remote control, No4, 2005, pp. 176-184.
[7] STEPANOV M.F., *Automatic solution of formalized problems of control theory*, Saratov: Saratovsky technique university, 2000.
[8] *MATLAB User's Guide*, MathWorks, 2001.
[9] ALEXANDROV A.G., *Finite - Frequency Identification and model validation of stable plant.* 14 th World Congress of IFAC, Preprints, Beijing, China, vol. H, 1999, pp.295 - 301.